Complex Variables, MATH 4337, Spring 2025

Homework 4: Section 2.13. Mappings, Section 2.14. Mappings of the Exponential Function, Section 15. Limits

Due Saturday, February 15 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the class notes, text book, or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`). The exercise numbers are based on the 9th edition of the textbook.

2.14.2. (a) Write the function $f(z)$ in the form $f(z) = u(x, y) + iv(x, y)$: $f(z) = \dfrac{\overline{z}^2}{z}$ where $z \neq 0$.

2.14.5. Find a domain in the z-plane whose image under the transformation $w = z^2$ is the square domain in the w-plane bounded by the lines $u = 1$, $u = 2$, $v = 1$, and $v = 2$.

2.18.2. Let a, b, and c denote complex constants. Use the definition of limit from Section 2.15 to prove the following.

(a) $\lim\limits_{z \to z_0} (az + b) = az_0 + b$.

2.14.8. [Graduate] Sketch the region onto which the sector $0 \leq r \leq 1$, $0 \leq \theta \leq \pi/4$ is mapped by the transformations **(a)** $w = z^2$, **(b)** $w = z^3$, **(c)** $w = z^4$.