Complex Variables, MATH 4337, Spring 2025

Homework 5: Section 15. Limits, Section 17. Limits Involving the Point at Infinity, Section 19. Derivatives

Due Saturday, March 1 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the class notes, text book, or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`). The exercise numbers are based on the 9th edition of the textbook.

2.18.9. Prove that if $\lim_{z\to z_0} f(z) = 0$ and if there exists a positive number M such that $|g(z)| \le M$ for all z in some neighborhood of z_0, then $\lim_{z\to z_0} f(z)g(z) = 0$. Give an "$\varepsilon$–$\delta$" proof.

2.18.10. Use Theorem 2.17.1 to show the following.

(a) $\displaystyle\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} = 4.$

(b) $\displaystyle\lim_{z\to 1} \frac{1}{(z-1)^3} = \infty.$

2.18.2. (Graduate) Let a, b, and c denote complex constants. Use the definition of limit from Section 2.15 to prove the following. **(b)** $\displaystyle\lim_{z\to z_0}(z^2+c) = z_0^2+c$. HINT: Choose $\delta = \min\left\{\frac{\varepsilon}{2|z_0|+1}, 1\right\}$.

2.20.8. (a) Use the technique of Example 2.19.2 to show that $f'(z)$ does not exist at any point of $\mathbb{C}$ for $f(z) = \mathrm{Re}(z)$.