Complex Variables, MATH 4337, Spring 2025

Homework 7: Section 24. Analytic Functions, Section 25. Examples, Section 27. Uniquely Determined Analytic Function, Section 29. The Exponential Function

Due Saturday, March 29 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the class notes, text book, or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`). The exercise numbers are based on the 9th edition of the textbook.

2.26.6(a). Use Theorem 2.23.A to verify that the function $g(z) = \ln r + i\theta$ (where $r > 0$ and $0 < \theta < 2\pi$) is analytic in the indicated domain of definition, and show that the derivative is $g'(z) = 1/z$. HINT: Theorem 2.23.A is the Cauchy-Riemann equations in polar coordinates.

2.26.7. Let a function f be analytic everywhere in a domain D. Prove that if $f(z)$ is real-valued for all z in D, then $f(z)$ must be constant throughout D.

3.30.6. Show that $|\exp(z^2)| \leq \exp(|z|^2)$.

3.30.8. Find all values of z which satisfy the given equation.

(a) $e^z = -2$.

(b) $e^z = 1 + i$.