Complex Variables, MATH 4337, Spring 2025

Homework 9: Section 37. Derivatives of Functions $w(t)$, Section 38. Definite Integrals of Functions $w(t)$

Due Saturday, April 12 at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the class notes, text book, or hypotheses. Use the notation and techniques described in the in-class hints. Do not discuss homework problems with others. If you have any questions, then contact me (`gardnerr@etsu.edu`). The exercise numbers are based on the 9th edition of the textbook.

4.42.1. Use rules in Calculus to establish the following rules when $w(t) = u(t) + iv(t)$ is a complex-valued function of a real variable t and $w'(t)$ exists.

(a) $\dfrac{d}{dt}[z_0 w(t)] = z_0 w'(t)$ where $z_0 = x_0 + iy_0$ is a complex constant.

4.42.2. Evaluate the following integral.

(d) $\displaystyle\int_0^\infty e^{-zt}\,dt$ where $\mathrm{Re}(z) > 0$.

4.42.3. Show that if m and n are integers, $\displaystyle\int_0^{2\pi} e^{im\theta}e^{-in\theta}\,d\theta = \begin{cases} 0 & \text{when } m \neq n \\ 2\pi & \text{when } m = n. \end{cases}$

4.42.4. **(Graduate.)** According to the definition of a definite integral of a complex-valued function of a real variable,

$$\int_0^\pi e^{(1+i)x}\,dx = \int_0^\pi e^x \cos x\,dx + i\int_0^\pi e^x \sin x\,dx.$$

Evaluate the two integrals on the right here by evaluating the single integral on the left and then by using the real and imaginary parts of the value found.