Section 6.73. Residues at Poles

Note. So far we have required a Laurent series to find a residue at a singular point. We now give a theorem which simplifies the procedure for finding residues when dealing with isolated singular points which are poles.

Theorem 6.73.1. An isolated singular point $z_0$ of a function $f$ has a pole of order $m$ if and only if $f$ can be written in the form $f(z) = \frac{\varphi(z)}{(z - z_0)^m}$ where $\varphi$ is analytic for $|z| < R_2$ for some $R_2 > 0$, and $\varphi(z_0) \neq 0$. Moreover,

$$\text{Res}_{z=z_0} f(z) = \begin{cases} 
\varphi(z_0) & \text{if } m = 1 \\
\frac{\varphi^{(m-1)}(z_0)}{(m-1)!} & \text{if } m \geq 2.
\end{cases}$$

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