Chapter 1. Complex Numbers

Study Guide

The following is a brief list of topics covered in Chapter 1 of Brown and Churchill’s *Complex Variables and Applications*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

**Section 1.1. Sums and Products.**
Definition of the field of complex numbers \( \mathbb{C} \) in terms of ordered pairs of real numbers, real and imaginary axes, \( i = (0, 1), i^2 = -1 \), “FOIL” holds in \( \mathbb{C} \).

**Section 1.2. Basic Algebraic Properties.**
Properties of addition and multiplication as given in Theorem 1.2.1, \( \mathbb{C} \) has no zero divisors, computation of \( 1/z \).

**Section 1.3. Further Properties.**
Addition with common denominators (Lemma 1.3.1), products of multiplicative inverses (Lemma 1.3.2), the Binomial Theorem (Theorem 1.3.2).

**Section 1.4. Vectors and Moduli.**
Modulus, the vector interpretation of complex numbers, the Triangle Inequality, other inequalities involving moduli (Corollary 1.4.1).

**Section 1.5. Complex Conjugates.**
Complex conjugate \( \overline{z} \), properties of conjugates (Theorem 1.5.1), the modulus of products and quotients (Theorem 1.5.2).

**Section 1.6. Exponential Form.**
Argument, principal value of the argument, Euler’s formula, exponential form \( z = re^{i\theta} \).

**Section 1.7. Products and Powers in Exponential Form.**
Products and quotients in exponential form (Theorem 1.7.1), powers in exponential form (Corollary 1.7.2), \( (\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta) \) (Corollary 1.7.3).

**Section 1.8. Arguments of Products and Quotients.**
Arguments of products (Lemma 1.8.1), arguments of quotients (Lemma 1.8.2), finding arguments.

**Section 1.9. Roots of Complex Numbers.**
nth roots of unity, \( e^{i\theta} \) is periodic, computing nth roots, distribution of nth roots around a regular
$n$-gon, the principal $n$th root.

**Section 1.10. Examples.**
Computation of $n$th roots.

**Section 1.11. Regions in the Complex Plane.**
Use of modulus to measure distance, definition of $\varepsilon$ neighborhood, and deleted $\varepsilon$ neighborhood, interior point (of a set), exterior point, boundary point, boundary, classification of a boundary point (Lemma 1.11.A), open set, closed set, closure of a set, connected open set, polygonal line, domain, region, bounded/unbounded set, accumulation point, closed sets contain their accumulation points (Lemma 1.11.B), isolated point.

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