Chapter 3. Elementary Functions

Study Guide

The following is a brief list of topics covered in Chapter 3 of Brown and Churchill’s *Complex Variables and Applications*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

**Section 3.29. The Exponential Function.**
Multiplying exponentials (Lemma 3.29.A), dividing exponentials.

**Section 3.30. The Logarithm Function.**
The “multiple-valued” function log $z$, the principal value Log $z$, relationship between log $z$ and Log $z$.

**Section 3.31. Branches and Derivatives of Logarithms.**
log $z$ as an analytic function, branch of the logarithm, branch cut, branch point.

**Section 3.32. Some Identities Involving Logarithms.**
Logarithm of a product (Lemma 3.32.A), logarithm of a quotient (Lemma 2.32.B), $z^n = e^{n \log z}$ (Lemma 3.32.C), $n$th roots in terms of logarithms (Lemma 3.32.D).

**Section 3.33. Complex Exponents.**
Definition of $z^c$ where $c \in \mathbb{C}$, branch of $z^c$, principal branch of $z^c$, principal value of $z^c$, derivative of $z^c$ (Theorem 3.33.A), examples, exponential function base $c$: $c^z$.

**Section 3.34. Trigonometric Functions.**
Definition of cos $z$ and sin $z$, derivatives of sin $z$ and cos $z$, addition formulas for sin and cos, double angle formulas, cofunction relationship between sin $z$ and cos $z$, real and imaginary parts of sin $z$ and cos $z$ (Lemma 3.34.A), the modulus of sin $z$ and cos $z$ and the fact that these are unbounded functions, zeros of a function, roots of a polynomial, zeros of sin $z$ and cos $z$ (Lemma 3.34.B), definitions and derivatives of the other trigonometric functions.

**Section 3.35. Hyperbolic Functions.**
Definition of cosh $z$ and sinh $z$, derivatives of cosh $z$ and sinh $z$ (Note 3.35.A), identities (Note 3.35.B), zeros of cosh $z$ and sinh $z$ (Note 3.35.C), definition and derivatives of the other hyperbolic trig functions.
Section 3.36. Inverse Trigonometric and Hyperbolic Functions.
Definition of \(\sin^{-1} z\) as a “multiple-valued function,” definitions of \(\cos^{-1} z\) and \(\tan^{-1} z\) as “multiple-valued functions,” derivatives of inverse trig and inverse hyperbolic trig functions.

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