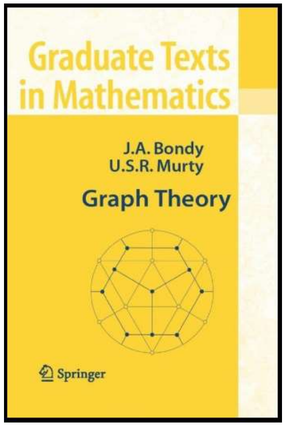
Graph Theory

Chapter 18. Hamilton Cycles

18.1. Hamilton and Nonhamiltonian Graphs—Proofs of Theorems



Theorem 18.1

Theorem 18.1. Let S be a set of vertices of a hamiltonian graph G. Then the number of connected component of $G - S$ satisfies $c(G - S) \leq |S|$. Moreover, if equality holds, then each of the $|S|$ components of $G - S$ is traceable, and every Hamilton cycle of G includes a Hamilton path in each of these components.

Proof. Let C be a Hamilton cycle of G. Then $C - S$ has at most $|S|$ connected components. Since C is a spanning subgraph of G then $G - S$ also has at most $|S|$ components. That is, $c(G - S) \leq |S|$, as claimed.

If equality holds and $G - S$ has exactly $|S|$ components, then $C - S$ also has exactly $|S|$ components (since this is the maximum number of components and $G - S$ has more edges than $C - S$). The connected components of $C - S$ are then spanning subgraphs of the connected components of $G - S$.

Theorem 18.1 (continued)

Theorem 18.1. Let S be a set of vertices of a hamiltonian graph G. Then the number of connected component of $G - S$ satisfies $c(G - S) \leq |S|$. Moreover, if equality holds, then each of the $|S|$ components of $G - S$ is traceable, and every Hamilton cycle of G includes a Hamilton path in each of these components.

Proof (continued). Since the components of $C - S$ are paths (in fact, spanning paths of the components of $C - S$) then the components of $G - S$ include Hamilton paths consisting of components of $C - S$. Therefore, the components of $G - S$ are traceable, as claimed. □