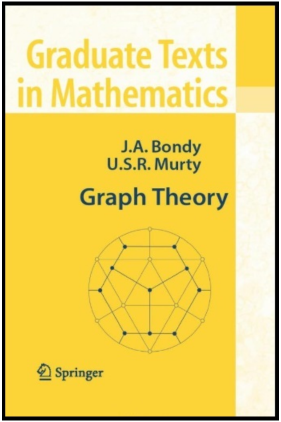
Graph Theory

Chapter 18. Hamilton Cycles

18.1. Hamiltonian and Nonhamiltonian Graphs—Proofs of Theorems



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Theorem 18.1

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Proof. Let C be a Hamilton cycle of G. Then $C - S$ has at most $|S|$ connected components. Since C is a spanning subgraph of G then $G - S$ also has at most $|S|$ components. That is, $c(G - S) \leq |S|$, as claimed.

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If equality holds and $G - S$ has exactly $|S|$ components, then $C - S$ also has exactly $|S|$ components (since this is the maximum number of components and $G - S$ has more edges than $C - S$). The connected components of $C - S$ are then spanning subgraphs of the connected components of $G - S$.

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Proof (continued). Since the components of $C - S$ are paths (in fact, spanning paths of the components of $C - S$) then the components of $G - S$ include Hamilton paths consisting of components of $C - S$. Therefore, the components of $G - S$ are traceable, as claimed. □