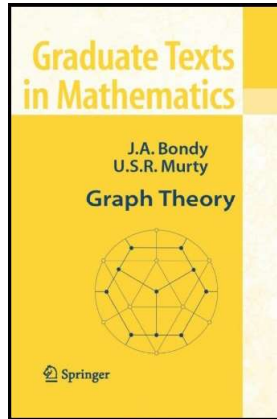


# Graph Theory

## Chapter 3. Connected Graphs

### 3.2. Cut Edges—Proofs of Theorems



## Proposition 3.2

**Proposition 3.2.** An edge  $e$  of a graph  $G$  is a cut edge of a graph  $G$  if and only if  $\{e\}$  belongs to no cycle of  $G$ .

**Proof.** Suppose  $e$  is a cut edge of graph  $G$ . Then  $c(G \setminus e) = c(G) + 1$  so graph  $G \setminus e$  has more than one connected component and hence  $G \setminus e$  has one more component than  $G$  so that there are different components of  $G \setminus e$ ,  $X$  and  $Y$ , such that one end of  $e$  is in  $X$  and the other end of  $e$  is in  $Y$ ; say  $e = xy$  where  $x \in X$  and  $y \in Y$ . Since  $X$  and  $Y$  are different components of  $G \setminus e$  then by Exercise 3.1.4 there is no  $(X, Y)$ -path in  $G \setminus e$ , so  $e$  lies in no cycle of  $G$  (or else the cycle with  $e$  deleted would be an  $(X, Y)$ -path in  $G \setminus e$ ).

Suppose  $e = xy$  is not a cut edge of  $G$ . Then  $x$  and  $y$  belong to the same component of  $G \setminus e$  (and  $c(G) = c(G \setminus e)$ ). So by Exercise 3.1.4 (with  $X = \{x\}$  and  $Y = \{y\}$  in the notation of the exercise) there is an  $xy$ -path  $P$  in  $G \setminus e$ . Then  $P + e$  is a cycle in  $G$  containing edge  $e$ . That is, if  $e$  is not a cut edge of  $G$  then  $e$  is in some cycle of  $G$ .  $\square$