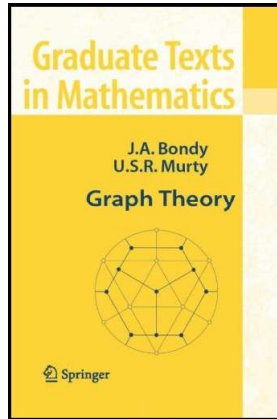


Graph Theory

Chapter 3. Connected Graphs

3.4. Connection in Digraphs—Proofs of Theorems



Theorem 3.6

Theorem 3.6. Let x and y be two vertices of a digraph D . Then y is reachable from x in D if and only if the output $\partial^+(X) \neq \emptyset$ for every subset X of V which contains x but not y .

Proof. Suppose that y is reachable from x . Then there is a directed (x, y) -path P in V . Let X be a subset of V which contains x but not y . Directed path P has its initial vertex in X and its final vertex in $V \setminus X$. Let u be the last vertex of P which belongs to X and let v be its successor in P . Then arc (u, v) is in $\partial^+(X)$ and so $\partial^+(X) \neq \emptyset$.

For the converse, suppose y is not reachable from x . Let X be the set of vertices which are reachable from x . Then $x \in X$ (since x is reachable from x by the trivial directed path from x to x) and $y \notin X$. So $y \in V \setminus X$ and $V \setminus X$ is nonempty. But no vertex of $V \setminus X$ is reachable from x , so the output $\partial^+(X) = \emptyset$. □