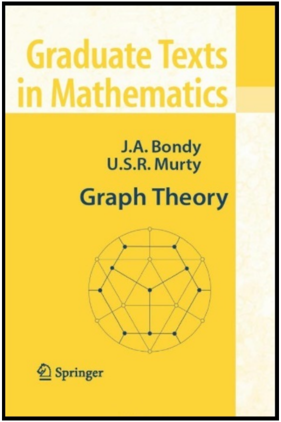
Graph Theory

Chapter 3. Connected Graphs

3.4. Connection in Digraphs—Proofs of Theorems



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Proof. Suppose that y is reachable from x. Then thee is a directed (x, y)-path P in V. Let X be a subset of V which contains x but not y. Directed path P has its initial vertex in X and its final vertex in $V \setminus X$. Let u be the last vertex of P which belongs to X and let v be its successor in P. Then arc (u, v) is in $\partial^+(X)$ and so $\partial^+(X) \neq \varnothing$.

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For the converse, suppose y is not reachable from x. Let X be the set of vertices which are reachable from x. Then $x \in X$ (since x is reachable from x be the trivial directed path from x to x) and $y \notin X$. So $y \in V \setminus X$ and $V \setminus X$ is nonempty. But no vertex of $V \setminus X$ is reachable from x, so the outcut $\partial^+(X) = \varnothing$. □

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