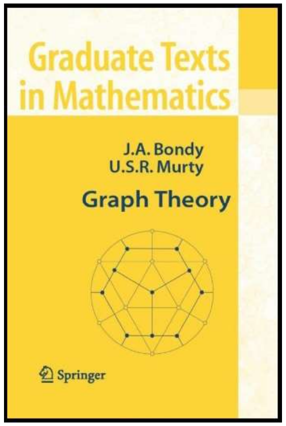
Graph Theory

Chapter 3. Connected Graphs

3.5. Cycle Double Covers—Proofs of Theorems



Proposition 3.8

Proposition 3.8. If a graph has a cycle covering in which each edge is covered at most twice, then it has a cycle double cover.

Proof. Let family $\mathcal{C} = \{C_1, C_2, \ldots, C_\ell\}$ be a cycle covering of a graph G in which each edge of G is covered at most twice. The symmetric difference

$$\triangle\{E(C) \mid C \in \mathcal{C}\} = (\cdots((C_1 \triangle C_2) \triangle C_3) \cdots \triangle C_\ell)$$

is the set of edges of G which are covered just once by $\mathcal{C}$ (associativity of the symmetric difference is shown in Exercise 2.6.2(a)). By Corollary 2.16 (and induction), the set of edges $\triangle\{E(C) \mid C \in \mathcal{C}\}$ induces an even subgraph of G. Then by Veblen's Theorem (Theorem 2.7), this even subgraph has a cycle decomposition $\mathcal{C}'$. So the edges of G that are covered only once by $\mathcal{C}$ are covered a second time by $\mathcal{C}'$ and so $\mathcal{C} \cup \mathcal{C}'$ is a cycle double cover of G. □