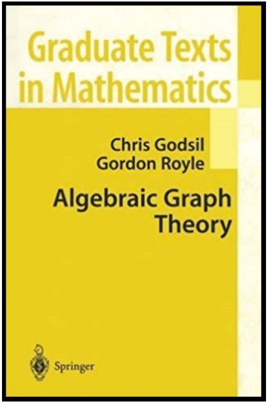
Graph Theory

Chapter 1. Graphs

1.3. Automorphisms—Proofs of Theorems



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Proof. Let $N(x)$ be the set of vertices adjacent to x ($N(x)$ is called the *open neighborhood* of x):

$$N(x) = \{y \in V \mid y \sim x\}.$$

Then $|N(x)|$ is the valency of x. Since g is an automorphism then (by the definition of isomorphism) $x \sim y$ if and only if $x^g \sim y^g$.

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$$N(x^g) = \{z \in V \mid z \sim x^g\}.$$

So $z \in N(x^g)$ if and only if $z = y^g$ for some $y \in N(x)$. That is, $g : N(x) \mapsto N(x^g)$.

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