

Graph Theory 1, MATH 5340, Fall 2020

Homework 2, 1.1. Graphs and Their Representations and

1.2. Isomorphisms and Automorphisms

Due Friday, September 4, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

1.1.7. n -CUBE. The n -cube Q_n (where $n \in \mathbb{N}$) is the graph whose vertex set is the set of all n -tuples of 0s and 1s, where two n -tuples are adjacent if they differ in precisely one coordinate (or “position”).

(a) Draw Q_1 , Q_2 , Q_3 , and Q_4 .

(b) Determine $v(Q_n)$ and $e(Q_n)$.

(c) Prove that Q_n is bipartite for all $n \in \mathbb{N}$.

1.1.9. Let $G[X, Y]$ be a bipartite graph.

(a) Prove that $\sum_{v \in X} d(v) = \sum_{v \in Y} d(v)$. Do so by summing the entries of the bipartite adjacency matrix in two ways.

(b) Prove that if $G = G[X, Y]$ is k -regular, with $k \geq 1$, then $|X| = |Y|$.

1.2.3. Let G be a connected graph. Prove that every graph which is isomorphic to G is connected.