

Graph Theory 1, MATH 5340, Fall 2020

Homework 3, 1.1. Graphs and Their Representations and

1.2. Isomorphisms and Automorphisms

Due Sunday, September 13, at noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

1.1.12. (a) Prove that if G is simple and $m > \binom{n-1}{2}$, then G is connected.

1.2.9. Prove that $\text{Aut}(G)$ is a group under the operation of function composition. HINT: Since we are not given that G is simple, then the elements of $\text{Aut}(G)$ are pairs of bijections $\theta : V(G) \rightarrow V(G)$ and $\varphi : E(G) \rightarrow E(G)$. So you need to define a binary operation $*$ on pairs (θ, φ) .

1.2.10. (a1) Show that, for $n \geq 2$, $\text{Aut}(P_n) \cong S_2$. HINT: You may assume that an automorphism π of a graph G maps each vertex v of G to a vertex $\pi(v)$ of G of the same degree (This is Exercise 1.2.1(a).)