Graph Theory 1, MATH 5340, Fall 2020

Homework 4, 1.1. Graphs and Their Representations and 1.2. Isomorphisms and Automorphisms, 1.3. Graphs Arising from Other Structures, 1.4. Constructing Graphs from Other Graphs

Due Sunday, September 20, at noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

1.1.17. Complement of a Graph.

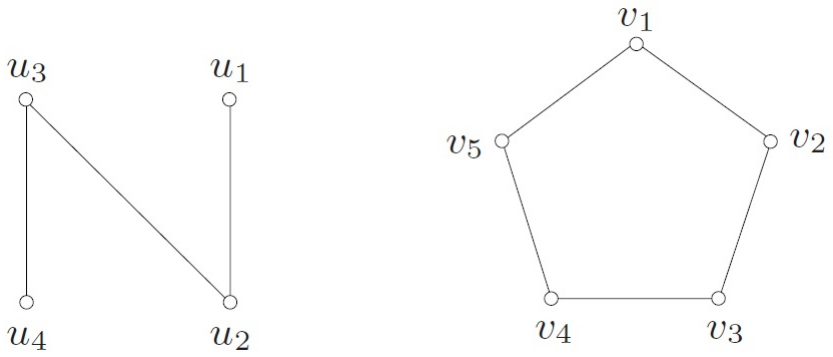
Let G be a simple graph. The *complement* $\overline{G}$ of G is the simple graph whose vertex set is V and whose edges are the pairs of nonadjacent vertices of G.

(b) Prove that if G is disconnected then $\overline{G}$ is connected. Is the converse true (that is, if G is connected then $\overline{G}$ is disconnected)?

1.2.16. Self-Complementary Graphs

A simple graph is *self-complementary* if it is isomorphic to its complement.

(a) Show that the graphs P_4 and C_5 of Figure 1.3 are self-complementary.



(b) Prove that every self-complementary graph is connected.

(c) Prove that if G is self-complementary, then $n \equiv 0$ or $1 \pmod 4$.

1.3.3. Show that the line graph of $K_{3,3}$ is self-complementary.

1.4.3. Prove that the Cartesian product is commutative. NOTE: For graphs G and H, we do not have, for example, $G \,\square\, H = H \,\square\, G$ because the vertex sets are different. But $G \,\square\, H$ and $H \,\square\, G$ are "naturally isomorphic."