

Graph Theory 1, MATH 5340, Fall 2020

Homework 6, 2.1. Subgraphs and Supergraphs, Solutions

Due Monday, October 5, at noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

2.1.1. Prove that the maximal connected subgraphs of a graph are its components. HINT: Use Exercise 1.4.1.

2.1.3. (a) Prove that if $m \geq n$ then G contains a cycle.

2.1.4. (a) Prove that every simple graph G contains a path of length δ .

2.1.15. Using Theorem 2.2 and the Cauchy-Schwarz Inequality (which, in $\mathbb{R}^n$, implies that

$$\left(\sum_{i=1}^n a_i b_i \right)^2 \leq \left(\sum_{i=1}^n a_i^2 \right) \left(\sum_{i=1}^n b_i^2 \right)$$

for real a_i, b_i where $1 \leq i \leq n$), prove that a simple graph G contains a quadrilateral if $m > \frac{1}{4}n(\sqrt{4n-3} + 1)$.