

Graph Theory 1, MATH 5340, Fall 2020

Homework 7, 2.2. Spanning and Induced Subgraphs,

2.3. Modifying Graphs

Due Sunday, October 18, at noon

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

2.2.7. Which grids have Hamilton paths, and which have Hamilton cycles? HINT: You may assume Theorem 4.7: A graph is bipartite if and only if it contains no odd cycle.

2.2.8. Give an example to show that the following simple procedure, known as the *Greedy Heuristic*, is not guaranteed to solve the Traveling Salesman Problem.

- ▷ Select an arbitrary vertex v .
- ▷ Starting with the trivial path v , grow a Hamilton path one edge at a time, choosing at each iteration an edge of minimum weight between the terminal vertex of the current path and a vertex not on the path.
- ▷ Form a Hamilton cycle by adding the edge joining the two ends of the Hamilton path.

2.2.11. Prove that if G is simple and connected, but not complete, then G contains an induced path of length two. You may assume Exercise 3.1.4 (to paraphrase): A graph G is connected if and only if for any two vertices u and v of G there is a path in G between u and v .

2.3.1. (a) Prove that $c(G/e) = c(G)$ for any edge e of a graph G . You may assume Exercise 3.1.4, which we paraphrase as: A graph G is connected if and only if for any two vertices x and y in G , there is a path in G joining x and y .