Graph Theory 1, MATH 5340, Fall 2022

Homework 3, 1.3. Graphs Arising from Other Structures, 1.4. Constructing Graphs from Other Graphs, Solutions

Due Saturday, September 10, at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

1.3.6. Let $H = (V, \mathcal{F})$ be a hypergraph. For $v \in V$, let $\mathcal{F}_v$ denote the set of edges of H incident to v. The *dual* of H is the hypergraph H^* whose vertex set is $\mathcal{F}$ and whose edges are the sets $\mathcal{F}_v$ where $v \in V$ (so edges $\mathcal{F}_u$ and $\mathcal{F}_v$ in H^* are adjacent if and only if vertices u and v are adjacent in H).

(a) How are the incidence graphs of H and H^* related?

1.3.9. Let G be a simple graph with incidence matrix $\mathbf{M}$.

(a) Prove that the adjacency matrix of the line graph $L(G)$ is $\mathbf{M}^t\mathbf{M} - 2\mathbf{I}$, where $\mathbf{I}$ is the $m \times m$ identity.

1.4.2. Prove that the rank over $GF(2)$ (the Galois field of order 2) of the incidence matrix of a simple graph G is $n - c$. NOTE: Since G is simple, the entries in the incidence matrix $\mathbf{M}$ are all 0 or 1. When we say "rank over $GF(2)$" we mean the dimension of the row space (or columns space) of $\mathbf{M}$ as a vector space with scalar field $GF(2)$. See Exercise 1.1.15.