

Graph Theory 1, MATH 5340, Fall 2024

Homework 11, 4.1. Forests and Trees, 4.2. Spanning Trees, 4.3. Fundamental Cycles and Bonds

“Due” Saturday, December 7, at 11:59 p.m., but DropBox will close on Thursday, December 12, at 3:20 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu). Use the same notation and terminology we used in class and given in the in-class hints.

4.1.3. A *saturated hydrocarbon* is a molecule C_mH_n in which every carbon atom (C) has four chemical bonds, every hydrogen atom (H) has one bond, and no sequence of chemical bonds forms a cycle. Show that, for any positive integer m , the molecule C_mH_n can exist if and only if $n = 2m + 2$.

4.2.1. Let G be a connected graph and e a link (i.e., an edge with distinct ends) of G .

(a) Describe a one-to-one correspondence between the set of spanning trees of G that contain e and the set of spanning trees of G/e .

4.3.6. (a) Let T be a spanning tree of a connected graph G . Prove that:

(i) The fundamental cycles of G with respect to T form a basis of its cycle space.