Graph Theory 1, MATH 5340, Fall 2024

Homework 2, 1.1. Graphs and Their Representations, 1.2. Isomorphisms and Automorphisms

Due Saturday, September 7, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu). Use the same notation and terminology we used in class and given in the in-class hints.

1.1.7. n-CUBE. The *n-cube* Q_n (where $n \in \mathbb{N}$) is the graph whose vertex set is the set of all n-tuples of 0s and 1s, where two n-tuples are adjacent if they differ in precisely one coordinate (or "position").

(a) Draw Q_1, Q_2, Q_3, and Q_4.

(c) Prove that Q_n is bipartite for all $n \in \mathbb{N}$.

1.2.3. Let G be a connected graph. Prove that every graph which is isomorphic to G is connected using our definition of connected.

1.2.9. Prove that $\mathrm{Aut}(G)$ is a group under the operation of function composition. HINT: Since we are not given that G is simple, then the elements of $\mathrm{Aut}(G)$ are pairs of bijections $\theta : V(G) \to V(G)$ and $\varphi : E(G) \to E(G)$. So you need to define a binary operation $*$ on pairs (θ, φ).