Graph Theory 1, MATH 5340, Fall 2024

Homework 4, 1.1. Graphs and Their Representation, 1.5. Directed Graphs

Due Saturday, September 21, at 11:59 p.m.

Write in complete sentences and in paragraphs!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu). Use the same notation and terminology we used in class and given in the in-class hints.

1.1.21. Eigenvalues of a Graph

Recall that the eigenvalues of a square matrix $\mathbf{A}$ are the roots of its characteristic polynomial $\det(\mathbf{A} - x\mathbf{I})$. An *eigenvalue* of a graph is an eigenvalue of its adjacency matrix. Likewise, the *characteristic polynomial* of a graph is the characteristic polynomial of its adjacency matrix. Prove the following.

(a) Every eigenvalue of a graph is real.

(b) Every rational eigenvalue of a graph is integral.

1.5.4. **(a)** Define the notions of vertex-transitivity and arc-transitivity for digraphs.

(b)(i) Prove that every vertex-transitive digraph is diregular.