Graph Theory 1, MATH 5340, Fall 2024

Homework 6, 2.4. Decompositions and Coverings

Due Saturday, October 26, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu). Use the same notation and terminology we used in class and given in the in-class hints.

2.4.1. Let e be an edge of an even graph G. Prove that G/e is even.

2.4.2. Even Directed Graphs.

A digraph D is *even* if $d^-(v) = d^+(v)$ for each vertex $v \in V$. Prove the following directed version of Veblen's Theorem (Theorem 2.7): A digraph D admits a decomposition into directed cycles if and only if it is even.

2.4.A. (a) Steiner Triple Systems.

A Steiner triple system of order n is a decomposition of K_n into isomorphic copies of K_3, denoted $STS(n)$. Prove that a necessary condition for the existence of a $STS(n)$ is $n \equiv 1$ or 3 (mod 6). NOTE: This condition is, in fact, also sufficient.