Graph Theory 1, MATH 5340, Fall 2024

Homework 7, 2.4. Decompositions and Coverings, 2.5. Edge Cuts and Bonds

Due Saturday, November 2, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not discuss homework problems with others. If you have any questions, then contact me (gardnerr@etsu.edu). Use the same notation and terminology we used in class and given in the in-class hints.

2.4.A. (b) A Steiner triple system admits an *automorphism* π, where $\pi : V(K_n) \to V(K_n)$ is a bijection, if for $K_3 = C_3 = x, y, z, x$ an element of the $STS(n)$ (that is, a subgraph in the decomposition of K_n) we have $\pi(C_3) = \pi(x)\pi(y)\pi(z)\pi(x)$ is also an element in the $STS(n)$. A $STS(n)$ is *cyclic* if it admits an automorphism consisting of a single cycle of length n. Show that a cyclic $STS(n)$ exists for $n = 7, 13, 19$, and 15.

2.5.1. (a) Prove Theorem 2.9: For any graph G and any subset X of V,

$$|\partial(X)| = \sum_{v \in X} d(v) - 2e(X).$$

(b) Prove Proposition 2.13: Let F_1 and F_2 be spanning subgraphs of a graph G, and let X be a subset of V. Then

$$\partial_{F_1 \triangle F_2}(X) = \partial_{F_1}(X) \triangle \partial_{F_2}(X).$$