Graph Theory 2, MATH 5450, Spring 2021

Homework 5, 13.1. Random Graphs

Due Wednesday, March 3, at 1:40

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

13.1.1. Let $G \in \mathcal{G}_{n,1/2}$. For a subset S of V, let $A(S)$ denote the event that S is a stable set of G (that is, no two vertices in S are adjacent in G; this is often called an *independent set*). Prove that if S and T are two distinct k-subsets of V, then $A(S)$ and $A(T)$ are independent if $|S \cap T| = 0$ or $|S \cap T| = 1$, and dependent otherwise.

13.1.2. Let V be an n-set. Consider the probability space (Ω, P), where Ω is the set of k-colourings $(V_1, V_2, \ldots, V_k)$ of V, all colourings being equiprobable (so the probability of a given vertex being assigned a particular colour is $1/k$ and a particular colouring of the n vertices has probability $1/k^n$). An element of this space is called a *random k-colouring* of V. Consider a random k-colouring of the vertices of a simple graph G. For an edge e of G, let A_e denote the event that the two ends of e receive the same colour. Prove that:

(a) For any two edges e and f of G, the events A_e and A_f are independent.

(b) If e, f, and g are the three edges of a triangle of G, the events A_e, A_f, and A_g are dependent.