Graph Theory 2, MATH 5450, Spring 2021

Homework 6, 13.2. Expectation

Due Friday, March 12, at 1:40

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

13.2.3. Let X be a nonnegative integer-valued random variable on finite probability space (Ω, P). Using the Cauchy-Schwarz Inequality, prove that $E(X^2)P(X \geq 1) \geq (E(X))^2$. HINT: Define $\Omega' = \{\omega \in \Omega \mid X(\omega) \geq 1\}$ and denote its elements as $\Omega' = \{\omega_1, \omega_2, \ldots, \omega_n\}$. "Cleverly" choose the components of two vectors $\vec{v}$ and $\vec{w}$.

13.2.7. **(a)** Let $G = (V, E)$ be a loopless graph. Consider a random 2-colouring of V (see Exercise 13.1.2). Prove that the expected number of edges of G whose ends receive distinct colors is $m/2$. HINT: Define random variable X as the number of edges of G whose ends receive distinct colours and show $E(X) = m/2$.

(b) Deduce that every (loopless) graph G contains a spanning bipartite subgraph F with $e(F) \geq \frac{1}{2}e(G)$. (This was also proved in Exercise 2.2.2(a).) HINT: From part (a), there must be at least one 2-colouring of V such that the number of edges whose ends are distinct colours is at least $m/2$.