Graph Theory 2, MATH 5450, Spring 2021

Homework 9, 10.1. Plane and Planar Graphs,

10.2. Duality

Due Friday, April 9, at 1:40

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

10.1.8. Consider a drawing $\tilde{G}$ of a (not necessarily planar) graph G in the plane. Two edges of $\tilde{G}$ *cross* if they meet at a point other than a vertex of $\tilde{G}$. Each such point is called a *crossing* of the two edges. The *crossing number* of G, denoted by $\mathrm{cr}(G)$, is the least number of crossings in a drawing of G in the plane.

(a) Prove that $\mathrm{cr}(G) = 0$ if and only if G is planar.

(b) Prove $\mathrm{cr}(K_5) = \mathrm{cr}(K_{3,3}) = 1$.

10.1.10. In Exercise 10.1.8 we have: "Consider a drawing $\tilde{G}$ of a (not necessarily planar) graph G in the plane. Two edges of $\tilde{G}$ *cross* if they meet at a point other than a vertex of $\tilde{G}$. Each such point is called a *crossing* of the two edges." A graph G is a *crossing-minimal* if $\mathrm{cr}(G \setminus e) < \mathrm{cr}(G)$ for all $e \in E$. Prove that every simple nonplanar edge-transitive graph is crossing minimal. Recall that a simple graph is *edge-transitive* if, for any two edges uv and xy, there is an automorphism α such that $\alpha(u)\alpha(v) = xy$ (see Exercise 1.2.17).

10.2.4. Prove that every planar graph G without cut edges has a cycle double cover. HINT: You may assume that a block of graph G is a single edge if and only if that edge is a cut edge of G (see Note 10.2.A).