

Graph Theory 2, MATH 5450, Spring 2023

Homework 11, 14.1. Chromatic Number,

15.2. The Four-Colour Theorem

Due Saturday, April 22, at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

14.1.9. Given any graph G , prove that there is an ordering of its vertices such that the Greedy Colouring Heuristic (Heuristic 14.3), applied to that ordering, yields a colouring with χ colours.

Note. This result is of no practical use in determining $\chi(G)$, since the construction starts with a χ -colouring which is used to generate the linear ordering.

15.2.2. By considering the partially coloured plane triangulation depicted below, show that the ‘double switching’ of colours of Kempe leads to an improper colouring. NOTE: This is a slight modification of Bondy and Murty’s Figure 15.10, which is *not* a triangulation.

