

Graph Theory 2, MATH 5450, Spring 2023

Homework 9, 10.5. Kuratowski's Theorem, 11.1. Colourings of Planar Maps

Due Saturday, April 8, at 11:59 pm

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

10.5.3. (a) Let F be a graph with maximum vertex degree at most three. Prove that a graph has an F -minor if and only if it contains an F -subdivision. HINT: To show the existence of the F -subdivision assuming an F -minor, we have by Note 10.5.A, that there is a partition (V_0, V_1, \dots, V_k) of the vertex set of G , where we have an isomorphic copy of F as the graph obtained from G by deleting V_0 and shrinking each induced subgraph $G[V_i]$, $1 \leq i \leq k$, to a single vertex.

11.1.2. Show that every even plane graph is 2-face-colourable. HINT: You may assume Exercise 10.2.10, which states that the dual G^* of an even plane graph G is bipartite.