Section 1.4. Constructing Graphs from Other Graphs

Note. In this section we describe a way to create a third graph from two given graphs G and H. In the presentation, G and H are assumed to be simple, though this restriction could be lifted.

Definition. Two graphs are *disjoint* if they have no vertex in common, and are *edge-disjoint* if they have no edge in common. The *union* of simple graphs G and H is the graph $G \cup H$ with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H)$. If G and H are disjoint, then $G \cup H$ is the *disjoint union* of G and H, denoted $G + H$.

Note. In Exercise 1.4.1 it is to be shown that every graph may be expressed uniquely (up to order) as a disjoint union of connected graphs.

Definition. For a graph G, the disjoint connected graphs which union to give graph G are the *connected components* (or just *components*) of G. The number of components is denoted $c(G)$.

Definition. The *intersection* of simple graphs G and H is the graph $G \cap H$ with vertex set $V(G) \cap V(H)$ and edge set $E(G) \cap E(H)$.

Note. If graphs G and H are disjoint then $G \cap H$ is the null graph. Figure 1.22 shows $G \cup H$ and $G \cap H$ for small graphs G and H.

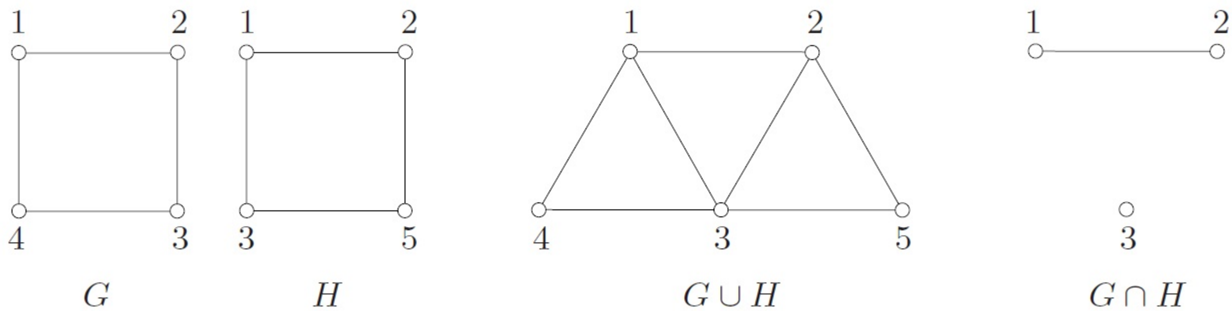

Figure 1.22

Definition. The *Cartesian product* of simple graphs G and H is the graph $G \square H$ whose vertex set is $V(G) \times V(H)$ and whose edge set is the set of all pairs $(u_1, v_1)(u_2, v_2)$ such that either $u_1u_2 \in E(G)$ and $v_1 = v_2$, or $v_1v_2 \in E(H)$ and $u_1 = u_2$

Note. For each $u_1u_2 \in E(G)$ and each $v_1v_2 \in E(H)$, there are four edges in $G \square H$, namely $(u_1, v_1)(u_2, v_1)$, $(u_1, v_2)(u_2, v_2)$, $(u_1, v_1)(u_1, v_2)$, and $(u_2, v_1)(u_2, v_2)$. Figure 1.23(a) gives $K_2 \square K_2$. The Cartesian product $P_m \square P_n$ of two paths is the $(m \times n)$-*grid* and is illustrated in Figure 1.23(b) for $m = 5$ and $n = 4$.

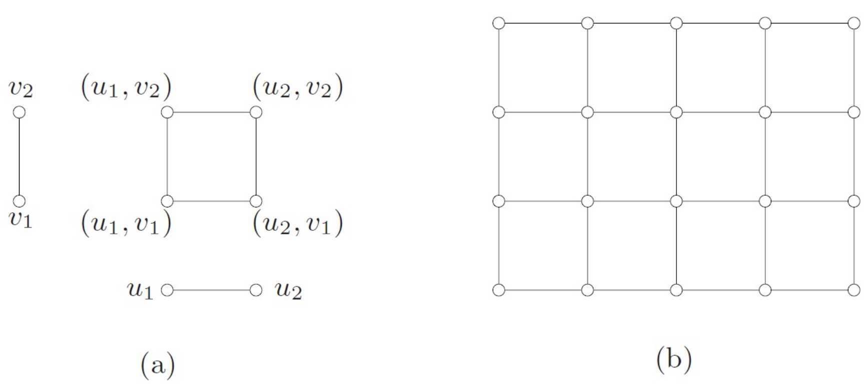

Figure 1.23

Definition. For $n \geq 3$, $C_n \,\square\, K_2$ is the *n-prism*. The 3-prism is the *triangular prism*, the 4-prism is the *cube*, and the 5-prism is the *pentagonal prism*.

Note. The 3-prism and 5-prism are illustrated in Figure 1.24.

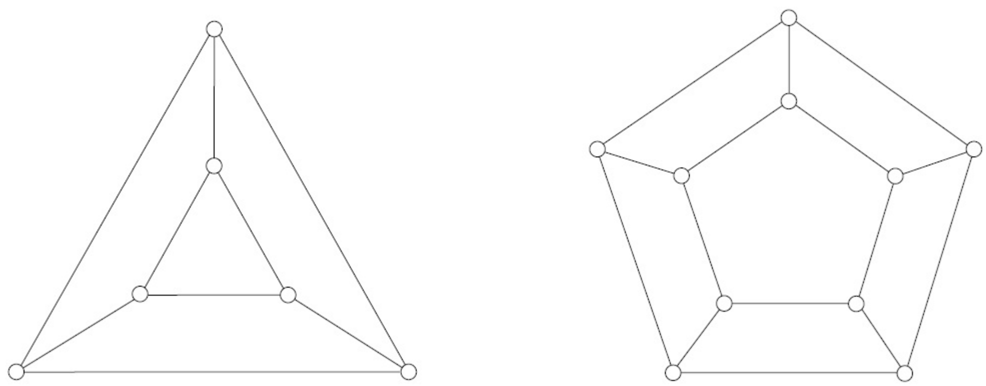

Figure 1.24

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