Section 3.2. Cut Edges

Note. We now define a cut edge of a graph (which is a special case of a bond), relate it to connectivity and classify cut edges in terms of cycles.

Note. Recall that the number of connected components of graph G is denoted $c(G)$. In Exercise 3.2.1 it is to be shown that if $e \in E(G)$ then either $c(G \setminus e) = c(G)$ or $c(G \setminus e) = c(G) + 1$.

Definition. A *cut edge* of a connected graph G is an edge of G such that $c(G \setminus e) = c(G) + 1$.

Note. In Exercise 3.2.2 it is to be shown that e is a cut edge of graph G if and only if $\{e\}$ is a bond of G. The cut edges of a particular graph are given in Figure 3.3:

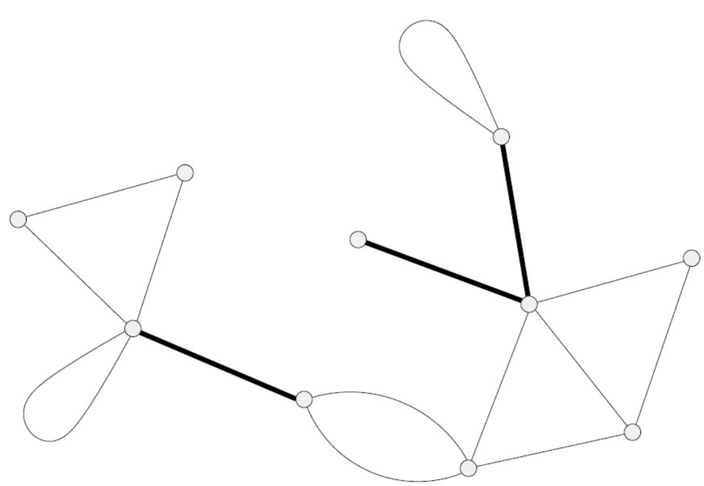

Figure 3.3

Note. A cut edge is also commonly called a *bridge*; this is maybe an even more common term than "cut edge." See my online notes for Introduction to Graph Theory (MATH 4347/5347) on Section 2.4. More Decompositions. However, in this class we stick with the term "cut edge," since we will use the term "bridge" in a different setting. In Section 10.4. Bridges we associate a "bridge" with a proper subgraph of a connected graph G (the bridge is a certain induced subgraph or G, not simply an edge of G).

Note. The following result gives a classification of cut edges.

Proposition 3.2. An edge e of a graph G is a cut edge of a graph G if and only if $\{e\}$ belongs to no cycle of G.

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