Chapter 5. Nonseparable Graphs

Section 5.1. Cut Vertices

Note. In Section 3.2 we defined a cut edge as an edge of a graph whose deletion results in an increase (by one) of the number of components of the graph. In this section we consider an analogous idea for vertices.

Definition. A *cut vertex* of a graph G is a vertex v such that $c(G - v) > c(G)$.

Note. Figure 5.1 gives a connected graph G and each of its cut vertices (the cut vertices are represented by solid vertices). Notice that, unlike with a cut edge, the deletion of a cut vertex can result in producing more than one "new" component.

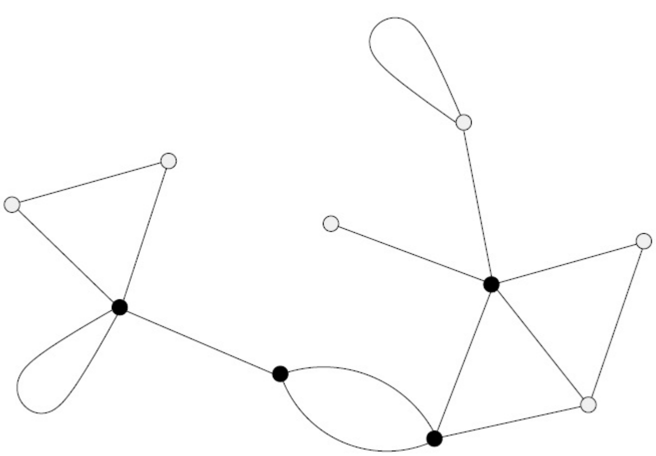

Figure 5.1

Note. In Exercise 3.1.4, it is to be shown that a graph is connected if and only if there is an (X, Y)-path for any two nonempty subsets of vertices X and Y. In particular, if every pair of vertices of graph G is connected by a path then G is connected. This leads us to consider properties of paths in a graph.

Definition. Two distinct paths in a graph are *internally disjoint* if they have no internal vertices in common.

Note. We now classify connected graphs that do *not* have cut vertices.

Theorem 5.1. A connected graph on three or more vertices has no cut vertices if and only if any two distinct vertices are connected by two internally disjoint paths.

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