Chapter 6. Tree-Search Algorithms

Note. The only algorithm we have encountered in the book so far is Fleury's Algorithm (Algorithm 3.3) which produces an Euler tour in an even connected graph (see Section 3.3. Euler Tours; in Theorem 3.4 we proved that Fleury's Algorithm works). In this chapter, we consider two algorithms to find a spanning tree in a connected graph (the Breadth-First Search, Algorithm 6.1, and the Depth-First Search, Algorithm 6.4, in Section 6.1), an algorithm to find a minimum spanning tree in an edge weighted graph (the Jarník-Prim Algorithm, Algorithm 6.9, in Section 6.2), and an algorithm to find a shortest path in an edge weighted graph (Dijkstra's Algorithm, Algorithm 6.12, in Section 6.3).

Note. Probably the most famous of these problems is finding a minimum spanning tree in an edge weighted graph. This is explored again in Chapter 8 in the Borůvka-Kruskal Algorithm (Algorithm 8.23). You might also see this in an Operations Research class. In particular, I have notes online for Introduction to Operations Research (not an official ETSU class, though a graduate level ETSU sequence, Operations Research 1 and 2 [MATH 5810/5820], does exist). See my online notes for Introduction to OR on Section 6.2. Minimal Spanning Tree Algorithm), where a brief version of the Borůvka-Kruskal Algorithm is given (along with a nice visual explaining the algorithm).

Note. This part of the book (namely, chapters 6, 7, 8, 20, and 21) contains material that is appropriate both for use in the graduate Graph Theory sequence

(MATH 5340 and 5450) and in the class Mathematical Modeling Using Graph Theory (MATH 5870). The modeling class was part of the online certificate program in "Mathematical Modeling in Biosciences Graduate Certificate." Classes for the short-lived certificate program were offered from fall 2013 to spring 2018. Mathematical Modeling Using Graph Theory was offered several times: spring 2015, spring 2016, spring 2017, and spring 2018 (taken by a total of 25 students, by my count). The catalog description for the modeling class was (from the ETSU 2017-18 Graduate Catalog):

> This course introduces the student to applied graph theory. Graph theoretical concepts will be approached as models for practical, real-world problems. The course will provide an introduction to graph modeling, integrated with applications based on emerging methods and needs. The emphasis is both on graphs as models—communication networks, for example—and on the algorithms used for obtaining information from those models.

A more appropriate description of the content based on Bondy and Murty's Chapters 6, 7, 8, 20, and 21:

> This course introduces the student to applied graph theory. Emphasis is on algorithms, especially tree-search algorithms, and the complexity of algorithms. Polynomial-time algorithms and the classes NP-complete and NP-hard are presented. The Max-Flow and Min-Cut Theorem is addressed in flows in networks. Integer flow and electrical networks are explored.

Revised: 5/24/2022