Chapter 1. Graphs
Study Guide

The following is a brief list of topics covered in Chapter 1 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 1.1. Graphs and Their Representation.
Graph, vertex, vertex set, edge, edge set, incidence function, vertices joined by an edge, ends of an edge, order and size of a graph, we largely consider finite graphs, drawings of graphs, vertices incident with an edge, adjacent vertices, neighbors, neighborhood $N_G(v)$, adjacent edges, loop, link, parallel edges, finite graph, simple graph, null graph, trivial graph, nontrivial graph, complete graph, empty graph, bipartite graph, bipartition, complete bipartite graphs, complement of a simple graph, star, path, cycle, length of a path, length of a cycle, k-path, k-cycle, even/odd path or cycle, connected graph, disconnected graph, intuitive idea of a planar graph/planar embedding, incidence matrix $\mathbf{M}_G$, adjacency matrix $\mathbf{A}_G$, adjacency list, bipartite adjacency matrix, degree of a vertex, isolated vertex, $\delta(G)$, $\Delta(G)$, The Handshaking Lemma, k-regular graph, regular graph, cubic graphs.

Section 1.2. Isomorphisms and Automorphisms.
Isomorphic graphs, isomorphism, complete graph K_n, complete bipartite graph $K_{m,n}$, path P_n, cycle C_n, automorphism, similar vertices, vertex-transitive graphs, asymmetric graph, the Peterson graph, automorphism group of graph G $\mathrm{Aut}(G)$, labeled simple graph, lower bound on the number of labeled simple graphs on n vertices.

Section 1.3. Graphs Arising from Other Structures.
Polyhedral graph, platonic graphs, set-system, hypergraph, vertices of a hypergraph, edges of a hypergraph, k-uniform hypergraph, geometric configuration and points and lines thereof, Fano plane, Desargue configuration, projective plane, incidence graph of a set system, incidence matrix of an incidence graph, intersection graph of a set system, line graph $L(G)$ of a loopless graph G, interval graph, dual of a hypergraph, self dual hypergraph, Fano plane (Figure 1.15(a)), incidence matrix of a hypergraph, Cayley graph, circulant.

Section 1.4. Constructing Graphs from Other Graphs.
Disjoint graphs, edge-disjoint graphs, union of simple graphs, disjoint union $G + H$, connected components, intersection of simple graphs, Cartesian product of two graphs, n-prism.

Section 1.5. Directed Graphs.
Directed graph, vertices of a directed graph, arcs of a directed graph, incidence function of a directed graph, join vertex u to vertex v, tail of an arc, head of an arc, ends of an arc, in-neighbors/out-neighbors of a vertex, digraph, strict digraph, underlying graph of a digraph $G(D)$, associated digraph $D(G)$, orientation of a graph, oriented graph, tournament, degree of a vertex, connected digraph, indegree/outdegree, $\delta^-(D)$, $\delta^+(D)$, $\Delta^-(D)$, $\Delta^+(D)$, k-diregular, source/sink vertex, directed path and cycle, converse of a digraph, Principle of Directional Duality, complete digraph, complete bipartite digraph, mixed graph, edges/arcs of a mixed graph, edge and arc incidence function of a mixed graph, complete mixed graph.

Section 1.6. Infinite Graphs.
Infinite graph, countable graph, square/triangular/hexagonal lattices, degree of a vertex in an infinite graph, one-way infinite path, two-way infinite path.

Section 1.7. Related Reading.
Graph Theory: 1736–1936 by Biggs, Lloyd, and Wilson; *Four Colors Suffice: How the Map Problem was Solved* by Wilson.

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