Chapter 10. Planar Graphs
Study Guide

The following is a brief list of topics covered in Chapter 10 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 10.1. Plane and Planar Graphs.
Embeddable in the plane/planar, plane graph, formal definition of a planar embedding in terms of homeomorphisms, points and line of a planar embedding $\tilde{G}$, homeomorphism, simple curve, simple closed curve, path, path connected/arcwise-connected, The Jordan Curve Theorem (Theorem 10.1), The Jordan Separation Theorem (Munkres' Theorem 61.3), interior and exterior of a simple closed curve, K_5 is not planar (Theorem 10.2), subdivision of graph G, a graph is planar if and only if every subdivision if planar (Proposition 10.3), stereographic projection of a sphere minus a point to the plane, a graph is embeddable in the plane if and only if it is embeddable in the sphere (Theorem 10.4).

Section 10.2. Duality.
Face and outer face of a plane embedding, boundary of a face, a face incident to vertices and edges, two faces adjacent, The Jordan Schönflies Theorem (Theorem 10.6), an edge separating a face, the degree of a face, subdividing a face with an edge, dual of a plane graph, plane dual of a graph, plane triangulation, a simple connected plane graph is a triangulation if and only if its dual is cubic (Proposition 10.11), congruence between duals of edge deleted graphs with contractions of contracted edges of duals (Propositions 10.12 and 10.13), the dual of a nonseparable plane graph is nonseparable (Theorem 10.14), directed plane dual, the relationship between cycles and bonds of a graph with bonds and cylces of the dual (Theorem 10.16).

Section 10.3. Euler's Formula.
Euler's Formula (Theorem 10.19), triangulations an simple planar graphs (Corollary 10.21), K_5 is nonplanar (Corollary 10.23), $K_{3,3}$ is nonplanar (Corollary 10.24).

Section 10.4. Bridges.

Bridge of a graph, vertices of attachment and internal vertices of a bridge, trivial bridge, k-bridge, equivalent bridges, segments of a bridge, two bridges which avoid each other, two bridges which overlap, two bridges which are skew, inner/outer bridge, in a plane graph inner bridges avoid one another and outer bridges avoit one another (Theorem 10.26), bridge-overlap graph, equivalent planar embeddings, unique embedding, a nonseparating cycle in a connected graph, facial cycle of a plane graph, every simple 3-connected planar graph has a unique planar embedding and a unique dual (Theorem 10.28 and Corollary 10.29).

Section 10.5. Kuratowksi's Theorem.

Kuratowski's Theorem (Theorem 10.30), minor of a graph, F-minor of a graph, Kuratowski minor of a graph, Kuratowski subdivision, minors of planar graphs are planar (Proposition 10.31), a graph is planar if and only if it has no Kuratowski minor (Wagner's Theorem, Theorem 10.32), Wagner's Theorem and Kuratowski's Theorem are equivalent (Note 10.5.C), every 3-connected nonplanar graph has a Kuratowski minor (Theorem 10.35), convex embedding, the proof of Wagner's Theorem, other classification results for planar graphs (Exercises 10.5.7, 10.5.8, 10.5.9), forbidden graphs in graphs embeddable on surfaces, The Planarity Recognition and Embedding (Algorithm 10.36).

Section 10.6. Surface Embeddings of Graphs.

Manifold, surface, cylinder, Möbius band, torus, Klein bottle, orientable and nonorientable surfaces, closed surface, homeomorphic, higher surfaces, adding a handle to a sphere, genus, sphere with k handles, adding a cross-cap, projective plane, cross-cap number, Classification Theorem for Closed Surfaces, John H. Conway's AIP proof of the classification theorem, Munkres' Classification Theorem (Theorem 77.5), fundamental domain, cellular embeddings, faces of a surface embedding, Euler characteristic of a closed surface, generalization of Euler's Formula to other surfaces (Theorem 10.37), using the generalization of Euler's Formula to show a graph is not embeddable in a surface, circular embedding, The Orientable Embedding Conjecture (Conjecture 10.40).

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