

Chapter 12. Stable Sets and Cliques

Study Guide

The following is a brief list of topics covered in Chapter 12 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 12.1. Stable Sets.

Stable set (independent set), maximum/maximal stable set, stability number $\alpha(G)$, edge covering, (vertex) covering, covering number $\beta(G)$, $\alpha(G) + \beta(G) = v(G)$ (Exercise 12.1.2), clique, clique number $\omega(G)$, strong product $G \boxtimes H$, application of stable sets in strong products (Example 12.1), stable set in a digraph, stability number of a digraph $\alpha(D)$, path partition, optimal partition, the number of paths in an optimal partition of digraph D $\pi(D)$, the Gallai-Milgram Theorem (Theorem 12.2), orthogonal directed path and stable set, orthogonal part partition and stable set, optimal path partitions in digraphs have orthogonal stable sets (Theorem 12.3 and its proof), Dilworth's Theorem (Theorem 12.5), kernel in a digraph, Richardson's Theorem (Theorem 12.6), semi-kernel in a digraph.

Section 12.2. Turán's Theorem.

Extremal graph with respect to a property, Turán graph and Exercise 1.1.11, Turán's Theorem (Theorem 12.3), the greedy algorithm to find a stable set in certain graphs (Exercise 12.2.4), combinatorial geometry, diameter of a set of points in the plane, sets of diameter 1 in the plane and points at a distance greater than $1/\sqrt{2}$ apart (Theorem 12.4) and an application (Note 12.2.A).

Section 12.3. Ramsey's Theorem.

Ramsey number $r(k, \ell)$, diagonal Ramsey numbers $r(k, k)$, bound on Ramsey numbers (Theorem 12.9), the known Ramsey numbers, Ramsey graph, upper bound on Ramsey numbers (Theorem 12.10 and Corollary 12.11), Lower bound on diagonal Ramsey numbers (Theorem 12.12), general Ramsey number $r(t_1, t_2, \dots, t_k)$, upper bound on general Ramsey number (Theorem 12.13 and Corollary 12.14), Schur's Theorem (Theorem 12.15).

Section 12.4. The Regularity Lemma.