Chapter 13. The Probabilistic Method
Study Guide

The following is a brief list of topics covered in Chapter 13 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 13.1. Random Graphs.
σ-field or σ-algebra of sets, finite probability space Ω, P), sample space, probability function, random graph in $\mathcal{G}_n$, random graph in $\mathcal{G}_{n,p}$, event, probability, two independent/dependent events, mutually independent events, random variables, indicator random variable, mutually independent random variables.

Section 13.2. Expectation.
Expectation of a random variable, linearity of expectation, crossing and crossing number, The Crossing Lemma (Lemma 13.1), the number of lines passing though points (Theorem 13.2), pairs of points at unit distance (Theorem 13.3), "almost surely," $\ll$ and $\gg$, Markov's Inequality (Proposition 13.4 and Corollary 13.5)), almost surely triangle free graphs (Note 13.2.B),almost surely upper bound on the stability number of a graph (Theorem 13.6).

Section 13.3. Variance.
Variance, Chebyshev's Inequality (Theorem 13.7), using Chebyshev's Inequality to establish an almost surely result (Corollary 13.8), covariance, almost surely triangle containing graphs (Note 13.3.A), almostsurely stability numbers (Theorem 13.9 and Corollary 13.10).

Section 13.4. Evolution of Random Graphs.
Monotone properties, threshold function of a monotone property, balanced graph, threshold function for the property of containing a given subgraph (Theorem 13.11).

Section 13.5. The Local Lemma.
Using probability to show the existence of certain properties in a finite probability space (Note 13.5.A), an event independent of a set of events, The Local Lemma (Theorem 13.12), dependency

digraph/graph, The Local Lemma—Symmetric Version (Theorem 13.14), Applications of The Local Lemma to 2-colourable hypergraphs (Theroem 13.15) and the existence of directed even cycles as subgraphs (Theorem 13.17), linear forest, linear arboricity, linear arboricity of a simple $2r$-regular graph of sufficiently large girth (Note 13.5.B, Lemma 13.18, and Theorem 13.19).

Section 13.6. Related Reading.

The papers of Erdős and Rényi, *Random Graphs* by Béla Bollobás, Frieze and Karonski's *Introduction to Random Graphs*, V. F. Kolchin's *Random Graphs*.

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