Chapter 15. Colourings of Maps
Study Guide

The following is a brief list of topics covered in Chapter 15 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 15.1. Chromatic Numbers of Surfaces.
Closed surface, chromatic number of a closed surface, Heawood's Inequality, embeddings on the torus, the Franklin graph, embeddings on the Klein bottle (Figure 15.2), Heawood's Conjecture, the Map Colour Theorem, embedding K_6 on the projective plane establishing its chromatic number as 6 (Figure 10.25(a)).

Section 15.2. The Four-Colour Theorem.
A smallest planar graph that is not colourable (Note 15.2.A), properties of a smallest counterexample to the Four-Colour Theorem (Proposition 15.2), a smallest counterexample has no vertex of degree four (Theorem 15.3), Kempe chains, Kempe interchange, other properties of a smallest counterexample (Theorem 15.4, Corollary 15.5, and Theorem 15.6), essentially 6-connected, Kempe's erroneous proof of the Four-Colour Theorem (Figure 15.6 and Exercise 15.2.2), Heawood's detection of the error, configuration, bounding cycle, reducible configuration, Birkhoff diamond (Figure 15.7), proof that the Birkhoff diamond is reducible (Theorem 15.7), unavoidable set of configurations, Appel and Hakin's discharging paper, Robertson et al.'s alternative proof of the Four-Colour Theorem, charge of a vertex, discharge a vertex, discharging algorithm, example of a discharging algorithm, Steinberg's Conjecture and Theorem 15.2.A, Steinberg's Conjecture on surfaces (Zhao's 2000 paper).

Section 15.3. List Colourings of Planar Graphs.
Near triangulation, list colourable near triangulations (Theorem 15.8 and Corollary 15.9), Grötzch's Theorem Theorem 15.10).

Section 15.4. Hadwiger's Conjecture.
Minor of a graph, Hajós graph (Figure 14.1), Hadwiger's Conjecture (Conjecture 15.11), a 4-

chromatic graph contains a K_4-subdivision (Theorem 15.12), sufficient conditions for a K_k-minor (Theorem 15.13 and Corollary 15.14), counterexamples to Hajós' Conjecture (Theorem 15.15).

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