Chapter 16. Matchings
Study Guide

The following is a brief list of topics covered in Chapter 16 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 16.1. Maximum Matchings.
Matching, matched vertices, perfect matching, maximum matching, matchable graph, maximal matching, matching number $\alpha'(G)$, The Maximum Matching Problem (Problem 16.1), The Assignment Problem (Problem 16.2)/The Scheduling Problem, M-alternating path, M-alternating cycle, M-augmented path, Berger's Theorem (Theorem 16.3).

Section 16.2. Matchings in Bipartite Graphs.
Hall's Theorem (Theorem 16.4; necessary and sufficient conditions for a matching cover all vertices or one partite set in a bipartite graph), system of distinct representatives, necessary and sufficient conditions for a perfect matching of a bipartite graph (Corollary 16.5), perfect matchings of regular bipartite graphs (Corollary 16.6), covering and minimum covering, covering number $\beta(G)$, minimal covering, König-Ore Formula, cardinalities of maximum matchings and minimum coverings (Proposition 16.7), cardinalities of maximum matchings and minimum coverings in bipartite graphs (The König-Egerváry Theorem, Theorem 8.32).

Section 16.3. Matchings in Arbitrary Graphs.
The number of odd components of graph G $o(G)$, relationship between matchings/subsets of V/set U of uncovered vertices (Lemma 16.3.A, equation (16.2)), there is no perfect matching of the Sylvester graph (Note 16.3.A), barrier of a graph, graphs with perfect matchings have $\varnothing$ as a barrier (Note 16.3.B), hypomatchable/factor-critical graphs, $\varnothing$ is a barrier of every hypomatchable graph (Lemma 16.8), essential/inessential vertices, connected graphs with no essential vertices are hypomatchable (Lemma 16.10), every graph has a barrier (The Tutte-Berge Theorem, Theorem 16.11), the matching number in terms of subsets of the vertex set (The Tutte-Berge Formula, Corollary 6.12).

Section 16.4. Perfect Matchings and Factors.

Necessary and sufficient conditions for a perfect matching (Tutte's Theorem, Theorem 16.13), 3-regular graphs without cut edges have perfect matchings (Petersen's Theorem, Theorem 16.14), f-factor, k-factor, reduction of the f-factor problem to the 1-factor problem (Note 16.4.A), T-join, the Weighted T-Join Problem, the Postman Problem (Exercise 16.4.22), the Minimum-Weight Matching Problem.

Section 16.5. Matching Algorithms.

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