

Chapter 17. Edge Colourings

Study Guide

The following is a brief list of topics covered in Chapter 17 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 17.1. Edge Chromatic Number.

k -edge colouring/colourable, edge chromatic number $\chi'(G)$, "colour classes" of edge colourings (Note 17.1.A), the Timetabling Problem (Example 17.1), colour represented/available at a vertex, colour available for an edge, ij -paths (Note/Definition 17.1.B), the edge chromatic number of a bipartite graph (Theorem 17.2).

Section 17.2. Vizing's Theorem.

Vadim Vizing's 1964 paper in Russian, the induction step in the proof of Vizing's Theorem (Lemma 17.3), illustration of the proof technique of Lemma 17.3 (Figure 17.2), Vizing's Theorem (Theorem 17.4), other proofs of Vizing's Theorem, the multiplicity of a graph, multigraph, Vizing's Theorem for multigraphs (Theorem 17.5), Class 1/Class 2 graphs, the problem of deciding if a graph is Class 1 or Class 2 is \mathcal{NP} -hard.

Section 17.3. Snarks.

Cubic graph, k -edge-connected, essentially $(k+1)$ -edge connected, snark, it suffices to establish the Cycle Double Cover Conjecture for snarks, Blasnuša snark (Figure 17.4), Isaac's infinite class of flower snarks.

Section 17.4. Coverings by Perfect Matchings.

Double covers (i.e., 2-coverings), Fulkerson's Conjecture (Conjecture 17.6), Claude Berge's weaker conjecture (Conjecture 17.7).

Section 17.5. List Edge Colourings.

List of colours for an edge $L(e)$, list edge colouring/colourable, k -list-edge colourable, list edge chromatic number $\chi'_L(G)$, $\chi'_L(G) \geq \chi'(G)$ (Note 17.5.A), the List Edge Colouring Conjecture (Con-

jecture 17.8), X -clique/ Y -clique, visualizing a line graph (Figure 17.7), Latin square (Exercise 17.5.1), kernels of orientations of line graphs (Theorem 17.9), “Galvin’s Theorem” for simple bipartite graphs (Theorem 17.10), extension of Theorem 17.10 to non-simple graphs (Exercise 17.5.3).

Section 17.6. Related Reading.

Total colouring, proper total colouring, total chromatic number $\chi''(G)$, the Total Colouring Conjecture (Conjecture 17.A; $\chi''(G) \leq \Delta + 2$), the total chromatic number of complete graphs (Behzad, Chartrand, and Cooper’s 1967 paper), Theorem 17.6.A ($\chi''(K_{n,n})$), Molloy and Reed’s result ($\chi''(G) \leq \Delta + 10^{26}$), Gary Chartrand and Ping Zhang’s *Chromatic Graph Theory*, Hian-Poh Yap’s *Total Colourings of Graphs*.

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