Chapter 2. Subgraphs
Study Guide

The following is a brief list of topics covered in Chapter 2 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 2.1. Subgraphs and Supergraphs.
Edge deletion, vertex deletion, edge-deleted subgraph, vertex-deleted subgraph, a subgraph is "contained in" a graph, copy of a graph, embedding, supergraph, proper supergraph, even graphs contain a cycle (Theorem 2.1), maximal/minimal elements of a family, maximal path, circumference of a graph, girth of a graph, acyclic graph, strict partially ordered set (or poset), irreflexive, antisymmetric, transitive, comparable/incomparable, chain/antichain.

Section 2.2. Spanning and Induced Subgraphs.
Spanning subgraph, edge addition, spanning subgraph, join of graphs, wheel/spokes, Hamilton path, Hamilton cycle, k-factor, Rédi's Theorem (Theorem 2.3), underlying simple graph, symmetric difference of two graphs, induced subgraph, edge induced subgraph, weight/weighted graph/weighted subgraph, The Traveling Salesman Problem.

Section 2.3. Modifying Graphs.
identify two vertices, contract an edge, split a vertex, subdivide an edge.

Section 2.4. Decompositions and Coverings.
Decomposition, path decomposition, cycle decomposition, isomorphic decomposition, even graph, Veblen's Theorem (Theorem 2.7), directed version of Veblen's Theorem (Exercise 2.4.2), cover/covering, uniform covering/k-cover, double cover, path covering, cycle covering, padding of a covering.

Supplement. Graph Decompositions, Coverings, and Packings.
Isomorphic decompositions, necessary conditions based on counting arguments, Steiner triple systems, construction techniques, directed triple systems, Mendelsohn triple systems, mixed triple systems, $\lambda > 1$ decompositions, coverings related to triple systems, packings, packings related to

triple systems, other decompositions/packing/coverings, auotmorphisms, difference methods.

Section 2.5. Edge Cuts and Bonds.
Edge cut/coboundary of a set of vertices, trivial edge cut, degree of a set of vertices in a loopless graph, even graphs and the cardinality of edge cuts (Theorem 2.10), symmetric difference of edge cuts (Proposition 2.11 and Corollary 2.12), bond of a graph, classification of edge cuts and bonds (Theorem 2.14), classification of bonds in terms of connectivity (Theorem 2.15), incut/outcut of a set of vertices in a digraph, outdegree/indegree of a set of vertices in a digraph, strongly connected digraph, directed bond.

Section 2.6. Even Graphs.
Even subgraph, the symmetric difference of two even subgraphs is an even subgraph (Corollary 2.16), classification of a set of edges as an even subgraph in terms of disjoint cycles (Theorem 2.17), the Galois field $GF(2)$, the vector spaces associated with a graph: edge space $\mathcal{E}(G)$, cycle space $\mathcal{C}(G)$, and bond space $\mathcal{B}(G)$; incidence vector.

Section 2.7. Graph Reconstruction.
hypomorphic graphs, graph reconstruction, "deck of cards," The Reconstruction Conjecture, reconstructible class, reconstructible parameter, the parameter $\binom{G}{F}$ and Kelly's Lemma, size and degree sequence are reconstructible parameters (Corollary 2.22), edge reconstructible graph/class/parameter, The Edge Reconstruction Conjecture, Edge Version of Kelly's Lemma, The Reconstruction Conjecture implies the Edge Reconstruction Conjecture, The Inclusion Exclusion Formula (Theorem 2.7.A), The Möbius Inversion Formula, Nash-Williams' Lemma (Lemma 2.26), a sufficient condition for edge reconstuctibility (Corollary 2.28).

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