

# Chapter 3. Connected Graphs

## Study Guide

The following is a brief list of topics covered in Chapter 3 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

### Section 3.1. Walks and Connection.

Walk, connect, initial/terminal vertex, segment of a walk, closed walk, trail, distance in a graph,  $(X, Y)$ -path, friendship graph, eigenvalues of a matrix of 1's (Lemma 3.1.A), The Friendship Theorem (Theorem 3.1).

### Section 3.2. Cut Edges.

Cut edge, necessary and sufficient conditions for an edge to be a cut edge (Exercise 3.2.2 and Proposition 3.2).

### Section 3.3. Euler Tours.

Königsberg Bridge Problem, tour, Euler tour, Euler trail, Eulerian graph, a necessary condition for a graph to be Eulerian (Lemma 3.3.A), Fleury's Algorithm (Algorithm 3.3 and Theorem 3.4), classification of Eulerian graphs (Theorem 3.5).

### Section 3.4. Connection in Digraphs.

Directed walk, directed trail/tour/path/cycle, reachable vertices, classification of reachable vertices (Theorem 3.6), strongly connected vertices, strong components of a digraph, directed Euler trail/tour, Eulerian digraph, classification of Eulerian digraphs (Theorem 3.7).

### Section 3.5. Cycle Double Covers.

Cycle double cover, a sufficient condition for a cycle double cover (Proposition 3.8), The Cycle Double Cover Conjecture (Conjecture 3.9), The Circular Embeddings Conjecture (Conjecture 3.10), small cycle double cover, The Small Cycle Double Cover Conjecture (Conjecture 3.11), The Oriented Cycle Double Cover Conjecture (Conjecture 3.12).