Chapter 9. Connectivity
Study Guide

The following is a brief list of topics covered in Chapter 9 of Bondy and Murty's *Graph Theory*, Graduate Texts in Mathematics 244 (Springer, 2008). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 9.1. Vertex Connectivity.

Local connectivity, k-connected, connectivity $\kappa(G)$, $\kappa(K_n) = n - 1$ (Note 9.1.A), connectivity of graphs with multiple edges (Note 9.1.B), xy-vertex-cut, set S separates x and y, local cut function, vertex cut, k-vertex cut, shrink a set of vertices, Menger's Theorem Undirected Vertex Version (Theorem 9.1), connectivity of a graph in terms of local connectivity (Theorem 9.2).

Section 9.2. The Fan Lemma.

The existence of (X, Y)-paths in k-connected graphs (Theorem 9.4), k-fan, The Fan Lemma (Proposition 9.5), in a 2-connected graph any two vertices lie on a common cycle (Note 9.2.A), k-connected graphs have cycles containing sets of size k (Theorem 9.6),

Section 9.3. Edge Connectivity.

Local edge connectivity, k-edge-connected, edge connectivity, Menger's Theorem Edge Version (Theorem 9.7), k-edge-cut, essentially $(k + 1)$-edge connected graphs and examples (Note 9.3.B), (x, y)-vertex cut in a digraph, Menger's Theorem Directed Vertex Version (Theorem 9.8), Menger's Theorem Arc Version (Theorem 7.16).

Section 9.4. Three-Connected Graphs.

S-component, marker edge, marked S-components, marked s-decomposition, 2-connected graphs with 2-vertex cuts (Theorem 9.9), decomposition tree, root and leaves of a decomposition tree, 3-connected components, decomposition trees and the Cycle Double Cover Conjecture, 3-connected graph G has an edge e such that G/e is 3-connected (Theorem 9.10), expansion of G at vertex v (illustrated in Figure 9.10), a condition that implies 3-connectivity of and expansion (Theorem 9.12).

Section 9.5. Submodularity.

Section 9.6. Gomory-Hu Trees.

Section 9.7. Chordal Graphs.

Chord, chordal graph, induced subgraphs of a chordal graph is chordal (Note 9.7.A), clique cut, a condition to guarantee a set is a clique (Theorem 9.19), simplicial decomposition of a chordal graph and Theorem 9.20, simplicial vertex, two nonadjacent vertices in a chordal graph (Theorem 9.21), simplicial order, chordal if and only id simplicial order (Corollary 9.22), chordal graph is and only if intersection graph of a family of subtrees of a tree (Theorem 9.23), tree representation, illustration of the ideas of this section.

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