Section 1.2. Subgraphs

Note. We complete our very brief survey of graph theory terms.

Definition. A subgraph of graph $X$ is a graph $Y$ such that $V(Y) \subseteq V(X)$ and $E(Y) \subseteq E(X)$. If $V(Y) = V(X)$ then $Y$ is a spanning subgraph of $X$. A subgraph $Y$ of $X$ is an induced subgraph if two vertices of $V(Y)$ are adjacent in $Y$ if and only if they are adjacent in $X$. See Figure 1.3.

![Figure 1.3. A spanning subgraph and an induced subgraph of a graph](image)

Definition. A subgraph of a given graph that is complete is called a clique. A set of vertices that induces an empty subgraph is an independent set. The size of the largest clique in a graph $X$ is denoted by $\omega(X)$, and the size of the largest independent set is denoted by $\alpha(X)$.

Definition. A path of length $r$ from $x$ to $y$ in a graph is a sequence of $r + 1$ distinct vertices starting with $x$ and ending with $y$ such that consecutive vertices are adjacent. If for any two distinct vertices in graph $X$ there is a path from one vertex to the other then graph $X$ is connected, otherwise $X$ is disconnected.
1.2. Subgraphs

**Note.** A graph $X$ is disconnected if and only if we can partition its vertices into two nonempty sets, $R$ and $S$, such that no vertex in $R$ is adjacent to a vertex in $S$. This is proved in Exercise 3.1.4 of Bondy and Murty’s *Graph Theory*, Graduate Texts in Mathematics #244 (2008, Springer).

**Definition.** If graph $X$ is disconnected such that nonempty sets $R$ and $S$ partition the $V(X)$ where no vertex of $R$ is adjacent to a vertex in $S$, then $X$ is the (edge) *disjoint union* of the two subgraphs induced be $R$ and $S$. A maximal connected induced subgraph of graph $X$ is a *connected component* of $X$.

**Definition.** A *cycle* is a connected graph where every vertex has exactly two neighbors. An *acyclic graph* is a graph which contains no cycles. A connected acyclic graph is a *tree* (an acyclic not necessarily connected graph is called a *forest*). A spanning subgraph containing no cycles is a *spanning tree*. A *maximal spanning forest* in graph $X$ is a spanning subgraph consisting of a spanning tree from each connected component.

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