# Chapter 2. Colorings of Graphs
## Study Guide

The following is a brief list of topics covered in Chapter 2 of Hartsfield and Ringel's *Pearls in Graph Theory: A Comprehensive Introduction* (Academic Press, 1994). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

**Section 2.1. Vertex Colorings.**
The Four Color Theorem, wheel with $n$ spokes $W_n$, color, coloring of a graph, chromatic number $\chi(G)$, subtraction of a vertex from a graph $G-v$, subtraction of an edge from a graph $G-e$, proper subgraph, critical graph (with respect to chromatic number), Theorem 2.1.1 (every critical graph is connected), Theorem 2.1.2 (graphs contain critical subgraphs), Theorem 2.1.3 (minimum degree of vertices in a critical graph), Theorem 2.1.4 (relationship between chromatic number, number of edges, and number of vertices), girth of a graph, Theorem 2.1.5 (graphs with given chromatic number and girth), definition of bipartite graph in terms of chromatic number, partite sets, distance between vertices, metric, Theorem 2.1.6 (classification of bipartite graphs in terms of even length cycles), diameter of a graph.

**Section 2.2. Edge Colorings.**
Adjacent edges, edge coloring, proper edge coloring, Theorem 2.2.1 (the number of colors in a proper edge coloring is at least maximum degree of any vertex), regular graph of degree $k$/$k$-regular graph, edge chromatic number, snark, Theorem 2.2.2 (Vizing's Theorem; edge chromatic number of a $k$-regular graph), Theorem 2.2.3 (edge chromatic number of $K_{2n}$ is $2n-1$), Theorem 2.2.4 (edge chromatic number of $K_{2n-1}$ is $2n-1$), 1-factor, $r$-factor.

**Section 2.3. Decompositions and Hamilton Cycles.**
Decomposition of a graph, Hamilton cycle, Hamilton path, Theorem 2.3.1 (Lucas' Theorem; Hamilton cycle decomposition of $K_{2n+1}$), difference methods, Theorem 2.3.2 (decomposition of $K_{2n}$ into Hamilton cycles and a 1-factor), Theorem 2.3.3 (decomposition of $K_{2n}$ into Hamilton paths), Theorem 2.3.4 (decomposition of $K_{2n}$ into paths of certain lengths), Theorem 2.3.5 (a snark has no Hamilton cycle).

**Section 2.4. More Decompositions.**

The "turning trick," Theorem 2.4.A (nonexistence of a path of length four decomposition of a cubic graph), bridge (cut edge), subgraphs which are banks of a bridge, components/connected components, Theorem 2.4.1 (classification of connected graphs as trees in terms of bridges), Theorem 2.4.2 (necessary and sufficient conditions for a decomposition of a connected graph into paths of length two).

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