Chapter 3. Circuits and Cycles

Study Guide

The following is a brief list of topics covered in Chapter 3 of Hartsfield and Ringel's *Pearls in Graph Theory: A Comprehensive Introduction* (Academic Press, 1994). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 3.1. Eulerian Circuits.
The first graph theory paper by Leonhard Euler, kónigsberg Bridge Problem, walk in a pseudograph, open walk, closed walk, trail, path in a pseudograph, length of walk, closed trail (or circuit), closed path (or cycle), triangle, loop, lune, degree of a vertex in a pseudograph, Eulerian circuit, Theorem 3.1.1 (Euler's Theorem; necessary condition for existence of an Eulerian circuit), Theorem 3.1.2 (necessary and sufficient conditions for the existence of an Eulerian circuit) and its proof, Theorem 3.1.4 (4-regular pseudograph has a decomposition into two 2-factors), Theorem 3.1.5 (Veblen's Theorem; necessary and sufficient conditions for a cycle decomposition), Eulerian trail, Theorem 3.1.6 (necessary and sufficient conditions for the existence of an Eulerian trail), "tracing puzzle," Theorem 3.1.7 (Listing's Theorem; Eulerian trails in a graph with $2h$ odd degree vertices).

Section 3.2. The Oberwolfach Problem.
The history of the Oberwolfach Problem, Oberwolfach Problem (in graph theoretic terms), use of the "turning trick" to give solutions to the Oberwolfach Problem (such as given in Figure 3.2.1), bridge, Theorem 3.2.1 (a regular graph of even degree has no bridge), Theorem 3.2.2 (a cubic graph that contains a bridge is not decomposable into three 1-factors), Petersen's Theorem (Theorem 3.2.3), Theorem 3.2.4 (cubic bridgeless graphs have decompositions into paths of length three), Theorem 3.2.5 (of Berge and Zhang; every 4-regular graph contains a 3-regular subgraph).

Section 3.3. Infinite Lattice Graphs.
The infinite graph L_2, Hamilton line, Eulerian line, one-way Eulerian trail, one-way Hamilton path.

Revised: 11/23/2022