Chapter 4. Extremal Problems

Study Guide

The following is a brief list of topics covered in Chapter 4 of Hartsfield and Ringel's *Pearls in Graph Theory: A Comprehensive Introduction* (Academic Press, 1994). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 4.1. A Theorem of Turan.
The order of a graph, size of a graph, induced subgraph, floor and ceiling functions, Theorem 4.1.A (the largest graph with chromatic number two is a complete bipartite graph), Theorem 4.1.1 ((the largest graph with chromatic number k is a complete k-partite graph), Theorem 4.1.2 (Turan's Theorem; largest graphs that contain no K_{k+1} subgraphs are k-partite), Theorem 4.1.2* (largest graph that contains no triangle is complete bipartite graph), proof of Turan's Theorem, Turan graph.

Section 4.2. Cages.
g-cage, Theorem 4.2.1 (the Petersen graph is the only 5-cage), Theorem 4.2.2 (the Heawood graph is the only 6-cage), other g-cages.

Section 4.3. Ramsey Theory.
Lemma 4.3.A (Theorem on Friends and Strangers; monochromatic triangles in K_6), biography of Frank P. Ramsey, Theorem 4.3.1 (Ramsey's Theorem), Lemma 4.3.B (monochromatic K_3 or K_4 in K_9), Ramsey number, known Ramsey numbers (Note 4.3.A), Theorem 4.3.2 (coloring complete graphs that give monochromatic complete subgraphs; Ramsey's Theorem is a special case), Lemma 4.3.C (coloring $K_{5,5}$ with two colors implies monochromatic $K_{2,2}$).

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