Chapter 5. Counting
Study Guide

The following is a brief list of topics covered in Chapter 5 of Hartsfield and Ringel's *Pearls in Graph Theory: A Comprehensive Introduction* (Academic Press, 1994). This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the proofs given in class and the homework problems.

Section 5.1. Counting 1-Factors.
There are $n!$ 1-factors in $K_{n,n}$ (Theorem 5.1.A), there are $\dfrac{n!}{2(n-k-1)!}$ different subgraphs of K_n isomorphic to path P_k (Theorem 5.1.B), combinations $\binom{n}{k}$, there are $n\dbinom{n-1}{3}$ different subgraphs of K_n of isomorphic to $K_{1,3}$ (Theorem 5.1.C), there are $\dfrac{(2h-1)!}{2^{h-1}(h-1)!}$ different 1-factors in K_{2h} (Theorem 5.1.D), the Hatcheck Problem and the Letter Problem (Note 5.1.A), derangement, the number of different 1-factors in $K_{n,n}$ minus a 1-factor (Theorem 5.1.E), the number of ways to answer a matching test with all answers wrong.

Section 5.2. Cayley's Spanning Tree Formula.
Cayley's Formula for the number of spanning trees in K_n (Theorem 5.2.1), the number of different sequences of length $n-2$ from a set of n symbols (Lemma 5.2.A), the Prüfer codes and spanning trees of K_n, proof of Cayley's Formula from J.A. Bondy and U.S.R. Murty's *Graph Theory with Applications* (Macmillan Press, 1976).

Section 5.3. More Spanning Trees.
The number of spanning trees in $K_{2,n}$ is $n2^{n-1}$ (Theorem 5.3.1), the number of spanning trees in $K_{3,n}$ is $n^2 3^{n-1}$ (Theorem 5.3.2), the number of spanning trees in the wheel W_n and recursion.

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