Section 1.5. Separation Axioms

Note. We now state the familiar separation axioms. These are given in Introduction to Topology (MATH 4357/5357). See my online notes for this class on 4.31. The Separation Axioms.

Definition 5.1. The separation axioms area:

($\mathbf{T}_0$) A topological space X is a *T_0-space* if for any two points $x \neq y$ there is an open set containing one of them but not the other.

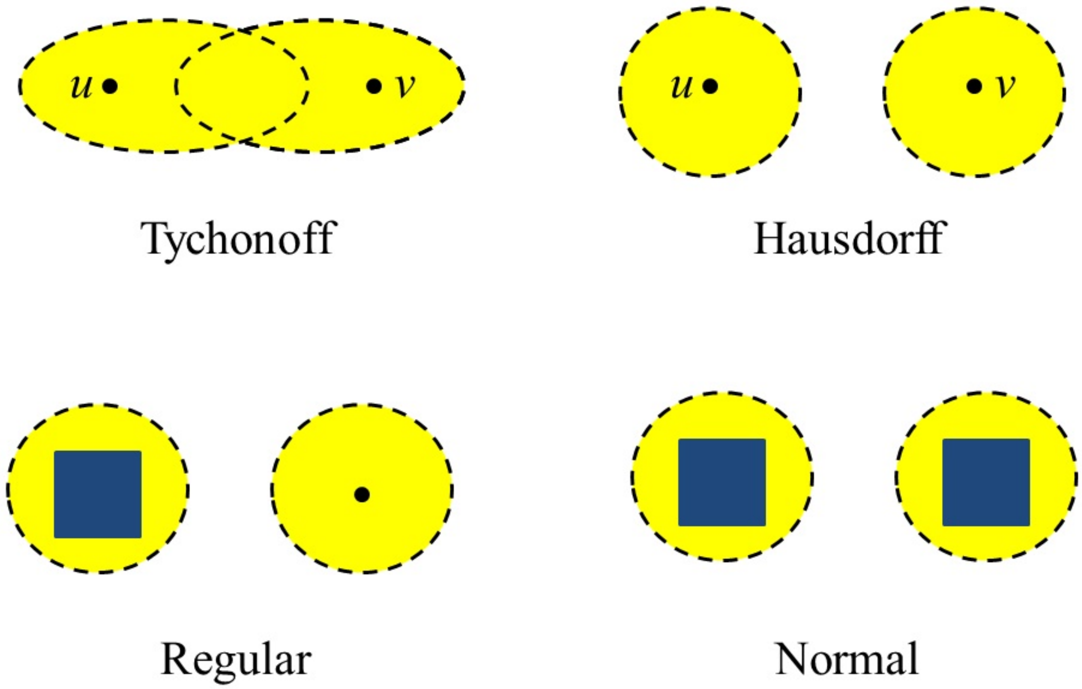
($\mathbf{T}_1$) A topological space X is a *T_1-space* or a *Tychonoff space* if for any two points $x \neq y$ there is an open set containing x but not y and another open set containing y but not x.

($\mathbf{T}_2$) A topological space X is a *T_2-space* or a *Hausdorff space* if for any two points $x \neq y$ there are disjoint open sets U and V with $x \in U$ and $y \in V$.

($\mathbf{T}_3$) A T_1-space X is called a *T_3-space* or a *regular space* if for any point x and closed set F not containing x there are disjoint open sets U and V with $x \in U$ and $F \subset V$.

($\mathbf{T}_4$) A T_1-space X is called a *T_4-space* or a *normal space* if for any two disjoint closed sets F and G there are disjoint open sets U and V with $F \subset U$ and $G \subset V$.

Note. We can illustrate the separation axioms as follows:



Proposition 5.2. A Hausdorff space is regular if and only if the closed neighborhoods of any point form a neighborhood basis of the point.

Corollary 5.3. A subspace of a regular space is regular.

Note. The exercises give a few properties of the separation axioms:

Exercise 1.5.5. A subspace of a Hausdorff space is Hausdorff.

Exercise 1.5.6. A Hausdorff space is normal if and only if for any sets U open and C closed with $C \subset U$ there is an open set V with $C \subset V \subset \overline{V} \subset U$.

Exercise 1.5.9. A metric space is normal.

Revised: 5/25/2021